STRONG EVAPORATION OF A LASER-HEATED WATER AEROSOL DROPLET

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Strong evaporation of a CO_2-laser-heated water aerosol droplet into the outer atmosphere is considered. The plots of the threshold values of laser radiation intensity and respective time of heating up to the droplet explosion vs the condensation coefficient have been obtained and analyzed. The effect of laser radiation intensity, starting from the threshold one, on the time of droplet heating up to its explosion and on the droplet mass at the instant of explosion was determined at different values of the condensation coefficient.

Calculation of heating and evaporation of aerosol particles by laser radiation evokes great interest. However, for the most part diffusional, diffusional-convective and gas-kinetic (sonic gas-dynamic) evaporation regimes have been extensively studied [1-4]. The present paper considers heating and strong (gas-dynamic) evaporation of water droplets into the outer atmosphere. The vapor Mach number varies from zero to unity in accordance with the time, droplet parameters, and intensity of heating.

Consider a spherical water droplet of radius $R_s >> \delta$, where δ is the Knudsen layer thickness. At time $t = 0$ the droplet, having the temperature $T = T_0$, begins to heat up by CO_2-laser radiation at a wavelength of $\lambda = 10.6$ μm. We shall assume that the radiation intensity I is fairly high, so that the influence of diffusive evaporation on the droplet temperature and radius can be neglected. The dynamics of heating and strong surface evaporation of a droplet, with the temperature dependence of thermophysical parameters taken into account, are described by the equations

$$C(T)\rho(T)\frac{\partial T}{\partial \tau}=\frac{1}{a_0R^2r^2}\frac{\partial}{\partial r}\left[k(T)r^2\frac{\partial T}{\partial r}\right]+\frac{C\rho r}{R}\frac{dR}{d\tau}\frac{\partial T}{\partial r}+\frac{3IK_nR_0}{4Ra_0}; \tag{1}$$

$$\frac{dR}{d\tau}=-\frac{R_0\rho_1U}{a_0\rho(T_s)},\quad 0<r\leqslant 1, \tag{2}$$

with boundary conditions

$$\frac{\partial T}{\partial r}=0 \text{ when } r=0; \tag{3}$$

$$\frac{\partial T}{\partial r}=-\frac{R_0R}{k(T_s)}\rho_1U\left[L(T_s)+C_p(T_1-T_s)+\frac{U^2}{2}\right] \text{ when } r=1 \tag{4}$$

and initial conditions

$$\left.\begin{matrix}T=T_0,\\ R=1\end{matrix}\right\} \text{ when } \tau=0. \tag{5}$$

The functions C(T), ρ(T), k(T), and L(T) are given in [4]. It is assumed that R_s is sufficiently small ($R_s \le 6$ μm), so that one can use the approximation of uniformly distributed sources having the intensity $3IK_n/(4R_s)$, where K_n is the factor of the effectiveness with which a droplet of radius R_s absorbs radiation. In calculations the function $K_n(R_s, T)$ was taken in the form [4]:

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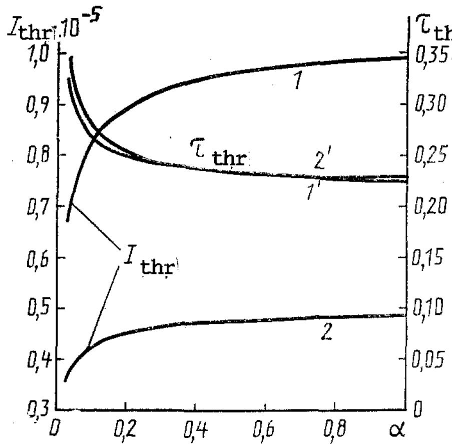

Fig. 1

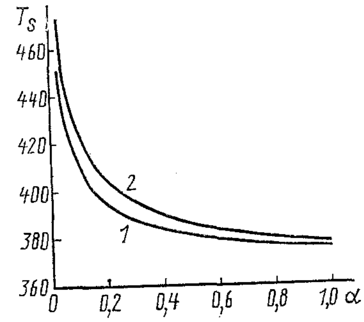

Fig. 2

Fig. 1. The threshold values of the intensity (1, 2) and of the nondimensionalized time of heating a droplet (1′, 2′) vs the condensation coefficient: 1, 1′) $R_0 = 4\ \mu m$; 2, 2′) 6 μm. I_{thr}, W/cm^2.

Fig. 2. Dependence of the quasisteady surface temperature on the condensation coefficient: 1) $IK_n = 2\cdot10^4\ W/cm^2$; 2) $3.2\cdot10^4$; T_s, K.

$$K_n = \exp\left[-0{,}2\left(\sqrt{n^2+\varkappa^2}-1\right)\right]\left[1-\exp\left(\frac{-8\pi\varkappa R_s}{\lambda}\right)\right], \tag{6}$$

where $n = 1.173$; $\varkappa = 3\varkappa_0 T' \int_0^1 r^2 dr/T(r)$; $\varkappa_0 = 0.073$; $T' = 283$ K.

At the assigned values of the specific heat ratio for vapor γ and for the condensation α and accommodation β coefficients, the quantities ρ_1, T_1, and U depend on T_s and on the Mach number M at the outer boundary of the Knudsen layer [5]

$$\frac{\rho_1}{\rho_s} = \varphi, \quad \frac{T_1}{T_s} = \psi. \tag{7}$$

For water $\beta = 1$ [6]. In this case [5], $\varphi = \varphi(M, \alpha, \gamma)$; $\psi = \psi(M, \gamma)$. The value of α for water is not yet known with assurance. According to [7], $0.3 \le \alpha \le 1$ at $T_s = 20°C$. The form of the functions $\psi(M, 1, \gamma)$ and $\varphi(M, \gamma)$ was found in [8]. For subsequent calculations the functions $\varphi(M, 1, \gamma)$ and $\varphi(M, \gamma)$ were borrowed from [8] with minor corrections made with consideration of [9]. In [5], formulae are presented which at $\beta = 1$ and after simple transformations, allow one to obtain an expression for $\varphi(M, \alpha, \gamma)$ in terms of $\varphi(M, 1, \gamma)$ and $\psi(M, \gamma)$:

$$\varphi(M, \alpha, \gamma) = \alpha\varphi(M, 1, \gamma)\left[\alpha + \varphi(M, 1, \gamma)(1-\alpha)M\sqrt{2\pi\gamma\psi(M, \gamma)}\right]^{-1}. \tag{8}$$

In [10] the relationship was obtained in a quasisteady approximation between the Mach number M on the Knudsen layer boundary and the ratio of the saturated vapor pressure $P_s(T_s)$ at the droplet surface temperature to the outer atmosphere pressure far from the droplet P_∞:

$$\frac{P_s}{P_\infty} = \left[\varphi(M, \alpha, \gamma)\psi(M, \gamma)\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}\right]^{-1}, \quad P_s > P_\infty,\ M \leqslant 1. \tag{9}$$

If the temperature within a certain region in the droplet attains the values $T \ge T_{expl}$, explosion of the droplet takes place [11]. At $P_\infty = 1$ atm, $T_{expl} = 584$ K [11]. In the calculations performed in the present work it was assumed (following [4]) that the droplet explodes when $T > T_{expl}$ in the region with $0 < r < 0.1$.

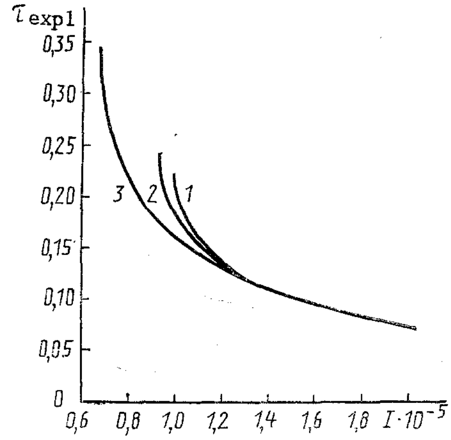

Fig. 3

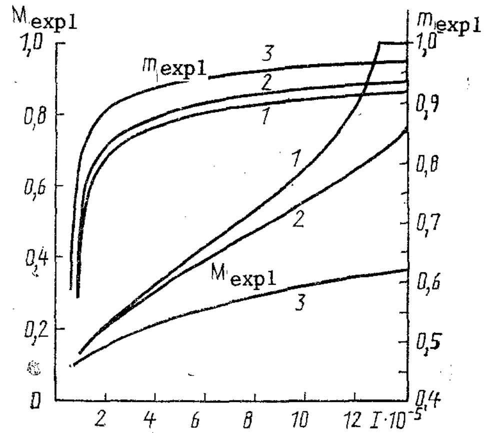

Fig. 4

Fig. 3. Dependence of the nondimensionalized time of droplet heating before the beginning of explosive evaporation on laser radiation intensity: 1) $\alpha = 1$; 2) 0.3; 3) 0.03. I, W/cm^2.

Fig. 4. Dependence of Mach number M_{expl} and of the relative mass of the droplet m_{expl} at the time of explosion on laser radiation intensity: 1) 1; 2) 0.3; 3) 0.03.

The plots of the threshold values of the intensity I_{thr} and of the corresponding nondimensionalized heating time τ_{thr} vs α at $P_\infty = 1$ atm, $T_0 = 285$ K, $R_0 = 4\ \mu$m (curves 1, 1') and $R_0 = 6\ \mu$m (curves 2, 2') are given in Fig. 1. As is seen from the figure, the condensation coefficient exerts the most perceptible influence on I_{thr} and τ_{thr} when $\alpha << 1$. Note that in the present case the increase of R_0 by 50% almost does not have any effect on the value of τ_{thr}, whereas the value of I_{thr} decreases almost by a factor of two.

The results obtained are explained as follows. At $I = I_{thr}$, by the time of droplet explosion, the temperature distribution is close to a quasisteady one. (Otherwise a further growth of temperature would have taken place.) In this case, the threshold intensity is determined from the relation [2]:

$$I_{expl}K_n \approx 8k_v(T_{expl} - T_s)/R_s,\ k_v = k(T_v),\ T_v = 373\ \text{K}. \tag{10}$$

Figure 2 demonstrates the influence of α on the quasisteady values of T_s obtained by solving the equation

$$\rho_1 U[L + C_p(T_1 - T_s) + U^2/2] = IK_n/4.$$

It is seen from the figure that the change in T_s becomes appreciable only when $\alpha << 1$.

The relative variation in R_s by the time of explosion is insignificant. Moreover, estimates show that it is also proportional to $T_{expl} - T_s$ (without regard for the thermal expansion of the droplet). Thus, the character of the dependence of I_{thr} on α, described above, is attributable to the corresponding dependence of $T_{expl} - T_s$ on α. It should also be noted that with $R_s \le 5\ \mu$m, the value of K_n becomes proportional to R_s. In such a case, $I_{thr} \sim R_0^{-2}$ in accordance with Eq. (10) and in agreement with the results given in Fig. 1.

In the cases considered, the time of heating the droplet center up to the temperature of explosive boiling-up without regard for evaporation is $\tau_1 \le \tau_{qu}$, where τ_{qu} is the time of the development of the quasisteady temperature distribution in the interior of the droplet. Consequently, when estimating the value of τ_{thr}, it is necessary to take into account both τ_{qu} and τ_1

$$\tau_{thr} \approx \tau_1 + \tau_{qu}.$$

According to [12], $\tau_{qu} \approx 0.1$. Using Eq. (10), one obtains

$$\tau_{thr} \approx (T_{expl} - T_0)/[8(T_{expl} - T_s) + 0,1. \tag{11}$$

The above numerical solution of the system of equations (1)-(9) has shown that R_0 exerts an extremely weak effect on τ_{thr}. This conclusion and the corresponding dependence of τ_{thr} on α are confirmed by estimation of Eq. (11).

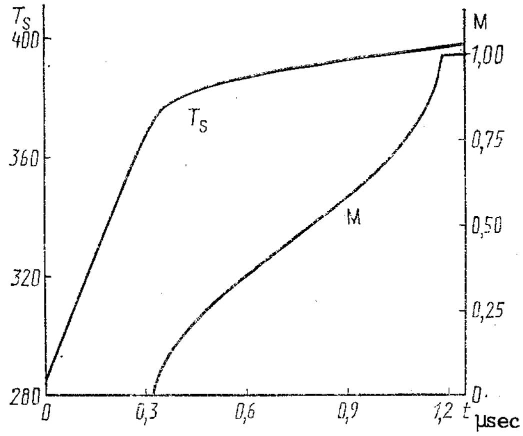

Fig. 5. The surface temperature and the vapor Mach number vs time: $\alpha = 1$, $P_\infty = 1$ atm, $I = 1.4 \cdot 10^6$ W/cm^2, $R_0 = 4\ \mu$m, $t_{expl} = 1.25\ \mu$sec.

In the present study, calculations of the explosive evaporation threshold characteristics at $T_{expl} = 590$ K, $\alpha = 1$, $P_\infty = 1$ atm, $T_0 = 285$ K, and $4 \leq R_0 \leq 6\ \mu$m were also carried out. The calculation results show that the effect of the increase in T_{expl} by 6 K is very small in complete agreement with Eq. (10). Such a variation in T_{expl} led to the rise of I_{thr} by 3% and to the fall of τ_{thr} by less than 1%.

Figure 3 presents the plots of the nondimensionalized time of droplet heating before the beginning of explosive evaporation τ_{expl} vs I at $\alpha = 1$, 0.3, and 0.03 (curves 1-3), starting from the threshold values for a droplet with $R_0 = 4\ \mu$m and $T_0 = 285$ K at $P_\infty = 1$ atm. In Fig. 4 the corresponding plots are given for the Mach numbers $M = M_{expl}$ and for the relative mass of the droplet m_{expl} (i.e., mass of the droplet normalized to its initial value) at the moment of droplet explosion. When $\alpha = 1$ and $I = I_{thr} = 0.99 \cdot 10^5$ W/cm^2, 40% of the initial mass of the droplet evaporate by the time of explosion, whereas at $I = 1.4 \cdot 10^6$ W/cm^2, only 8% evaporates. The double excess of the intensity I over the threshold value leads to almost a double decrease in the mass which evaporates by the time of explosion.

As is seen from Fig. 3, the value of α influences the value of τ_{expl} only near I_{thr}. This is due to the fact that τ_{expl} decreases rapidly with the rise in I, whereas with $\tau_{expl} < 0.1$ the surface evaporation has no time to appreciably influence the temperature of the most heated region of the droplet near the center. It follows from the results of calculations given in Fig. 4 that the value of the condensation coefficient exerts an appreciable effect on the value of M_{expl}. It should be noted that at $\alpha = 1$, $M_{expl} = 1$ starting from $I = 1.29 \cdot 10^6$ W/cm^2, whereas at $\alpha = 0.3$ and 0.03 and $I = 1.29 \cdot 10^6$ W/cm^2, the evaporation is still subsonic and $M_{expl} = 0.69$ and 0.35, respectively.

Note that the values of I_{thr} found by the present author with the aid of the subsonic gas-dynamic scheme are close to the threshold values obtained at $\alpha = 1$ and $\alpha = 0.036$ in [3] on the basis of the diffusional-convective approximation which is valid for $M \ll 1$. The corresponding differences at $R_0 = 6$ and 4 μm do not exceed 20%.

The nearness of the results obtained in the present work to the results of [3] means that even though M_{thr} lies close to the boundary of the region of the admissible values of M_{thr} in the subsonic scheme ($M_{thr} \approx \delta/R_s$), one may use both the diffusional-convective approximation [3] and a much less cumbersome gas-dynamic scheme for determining the threshold radiation intensity and time of heating water droplets with the radii of several microns at $P_\infty = 1$ atm. At the same time, for calculating the subsonic evaporation at radiation intensities greatly exceeding the threshold ones, the gas dynamic approximation should be used, as can be seen from Fig. 4.

Figure 5 presents the plots of the surface temperature and vapor Mach number vs time up to the instant of droplet explosion obtained by solving the system of Eqs. (1)-(9).

If follows from Fig. 3 that $\tau \ll 1/4$ when $I \gg I_{thr}$ at the stage of preexplosive evaporation. As noted above, in this case the spherical symmetry of the heat conduction equation does not influence the temperature field within the droplet. The sphericity of the problem reveals itself through the Mach number which enters into boundary condition (4) and which is determined with the help of Eq (9). But Eq. (9) was obtained for quasisteady evaporation. In the case of subsonic evaporation, this imposes a limitation from above on the admissible values of R_0. The characteristic time of variation in the gas-dynamic parameters on the droplet surface should be much in excess of $2R_0/U_0$. The value of the radius influences only the intensity of volumetric sources $3IK_n/(4R_s)$. In this case, when $R_s \leq 5\ \mu m$, this influence weakens significantly, since K_n becomes proportional to R_s.

CONCLUSIONS

The paper considers a strong evaporation of a CO_2-laser-heated water aerosol droplet into the outer atmosphere. The plots of the threshold values of the laser radiation intensity and corresponding time of droplet heating up to its explosion against the condensation coefficient have been obtained and analyzed. At different values of the condensation coefficient the dependence of the time of droplet heating up to explosion and its mass at the time of explosion on the laser radiation intensity, starting from the threshold one, has been found for different values of the condensation coefficient. It is shown that both the diffusional-convective approximation and the gas-dynamic scheme can be employed when determining the threshold intensity for water droplets with the radii of several microns at the atmospheric pressure. At the same time, in order to calculate the pre-explosive stage of evaporation at the intensities greatly exceeding the threshold one, the gas-dynamic approximation should be used.

NOTATION

R_s, droplet radius; R_0, initial radius of droplet, $R = R_s/R_0$; r, spherical coordinate nondimensionalized by R_s; T_s, droplet surface temperature; T_0, initial temperature; T, temperature within a droplet at the point with coordinate r; C, ρ, k, and L, heat capacity, density, thermal conductivity, and latent heat of vaporization of condensed phase; a_0, thermal diffusivity at T = 273 K; t, time; $\tau = ta_0/R_0^2$; I, laser radiation intensity; λ, radiation wavelength; n and $\varkappa$, mean values of the real and imaginary parts of the refraction index; ρ_1, T_1, U, and M, density, temperature, velocity, and Mach number of vapor at the Knudsen layer boundary; C_p, vapor heat capacity; ρ_s and P_s, saturated vapor density and pressure at temperature T_s; P_∞, outer atmosphere pressure far from a droplet; m, ratio of droplet mass to its initial value; T_{expl}, explosive boiling-up temperature; t_{expl}, τ_{expl}, M_{expl}, and m_{expl}, values of t, τ, M, and m at time of droplet explosion; I_{thr} and τ_{thr}, minimum value of I and corresponding value of τ at which droplet explodes.

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